Direct Numerical Simulation of the Mixing Layer past Serrated Nozzle Ends

Andreas Babucke, Markus J. Kloker and Ulrich Rist

Abstract The effect of serrations on the mixing layer past a thin splitter plate is investigated using spatial direct numerical simulation (DNS) with direct sound computation. Two different geometries are considered which yield a spanwise deformation of the Kelvin-Helmholtz rollers, streamwise vortices and subsequent breakdown of large-scale coherent structures, strongly affecting the noise emission. The results reveal that the spanwise extent of the serration is the driving parameter for sound reduction while its actual shape is less important.

1 Introduction and Numerical Method

Noise reduction is an important issue in aviation. Especially jet noise is one of the major acoustic sources of an aircraft. Modification of the trailing edge shape is a recent approach to reduce jet noise [3], often explained by an increased mixing behind the nozzle end. However the underlying physical mechanisms are not yet well understood.

The considered flow is made of two laminar boundary layers with the same freestream temperatures and $Ma_I = 0.8$ and $Ma_{II} = 0.2$ above and below the splitter plate, respectively. The Reynolds number $Re = 1000$ is based on the displacement thickness $\delta_{1,I}$ of the upper boundary layer at the inflow. Two different types of serrations are considered: a non-symmetric serration (figure 2) and a rectangular notch with half the spanwise extent (figure 3). The depth of the serrations is 10 for both cases being roughly three times the boundary-layer thickness δ_{99} at the trailing edge. The extent in z-direction defines the fundamental spanwise wavenumber $\gamma_0 = 0.2$. The flow is forced at the inflow of the upper boundary layer with a 2-d Tollmien-Schlichting wave $(1,0)$ of frequency $\omega_0 = 0.0688$ and $|\hat{u}| = 5 \cdot 10^{-3}$, and a low-amplitude oblique wave $(1,\pm1)$ with $|\hat{u}| = 5 \cdot 10^{-4}$. In the following, distur-

Andreas, Babucke, Markus Kloker, Ulrich Rist
IAG Universität Stuttgart, Germany, e-mail: [babucke]/[kloker]/[rist]@iag.uni-stuttgart.de

bances are denoted as (h,k), where h and k are the multiples of the fundamental frequency ω_0 and the fundamental spanwise wavenumber γ_0, respectively.

The simulations are carried out using the DNS-code NS3D which solves the 3-d unsteady compressible Navier-Stokes equations, see [1] for details. The serration is implemented by defining a region without wall at the respective grid points. There, the spatial derivatives in normal direction are recomputed, using values from the domain at the other side of the splitter plate as well. This is done by explicit finite differences of 8^{th} order which are designed such that their numerical properties closely match those of the 6^{th}-order compact scheme used in the rest of the domain. With spanwise filtering due to the spectral ansatz being applied only to the time derivatives of the flow variables, the no-slip condition is held completely at the wall.

2 Results

For the reference case with the straight trailing edge, shown in figure 1, the flow is dominated by 2-d Kelvin-Helmholtz vortices. As shown in figure 2, the non-symmetric serration causes a spanwise deformation of the Kelvin-Helmholtz vortices similar to the experimental results of Kit et al. [4]. The resulting shape of the vortex corresponds to the trailing-edge shape. At the location of the strongest spanwise gradients, longitudinal vortex tubes are generated which are directed towards the center of the notch, see figure 4. Further downstream these interact with the spanwise rollers leading to a breakdown to small scale structures.

In case of the rectangular serration with half the spanwise extent (figure 3) the initial Kelvin-Helmholtz vortex at $x \approx 50$ is modulated along the spanwise direction as well. Due to the double number of the serrations, the first roller is modulated with spanwise wavenumber $2 \cdot \gamma_0$, here. As shown in figure 5, longitudinal vortices exist as well but they are directed much more downward, compared to the wider non-symmetric notch. Although more streamwise vortex tubes are generated, the 2-d structure of the spanwise rollers is preserved longer. Yet the dominance of small-scales occurs for both cases at $x \approx 120$.

The spectral decomposition in figure 6 a) shows that the non-symmetric serration generates steady spanwise modes $(0,k)$ up to $|\hat{v}|_{max} = 6 \cdot 10^{-3}$ at the trailing edge. With increased amplitude, they interact with the 2-d TS-wave $(1,0)$ and its higher harmonics, generating oblique modes $(h,1)$, see figure 6 b).

With doubled number of serrations, only steady disturbances with spanwise wavenumbers $\geq 2\gamma_0$ are introduced by the rectangular serrations (figure 7 a)). Mode $(0,1)$ stays on the lower level defined by the interaction of $(1,0)$ and $(1,\pm1)$ in the upper boundary layer. Accordingly, oblique waves with wavenumber $2\gamma_0$ are generated past the trailing edge and mode $(1,\pm1)$ is smaller by one order of magnitude as given in figure 7 b). Yet at the end of the integration domain, it reaches a similar level as for the non-symmetric case due to amplification inside the mixing layer. High-level 2-d waves $(1,0)$-$(3,0)$ show a similar behavior for both serrations which is also the case for their subharmonics (not shown here).

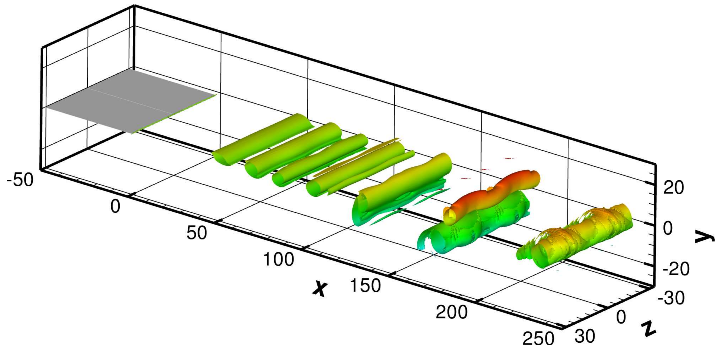

Fig. 1 Flow structures for the reference case with straight trailing edge, visualized by the isosurface $\Lambda_2 = -0.001$ along two spanwise periods. Grey scales indicate the wall-normal distance.

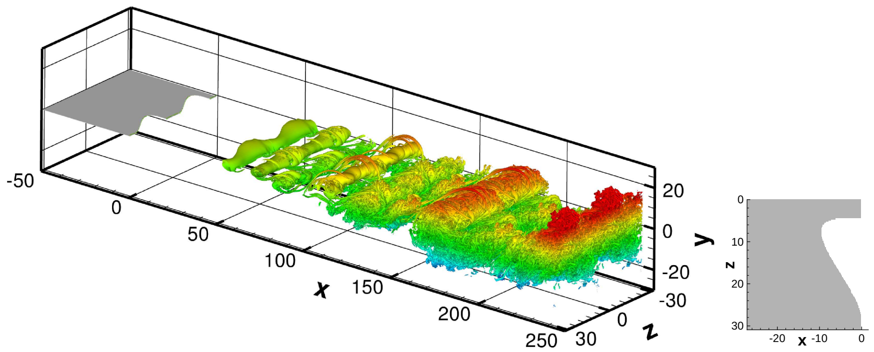

Fig. 2 Snapshot of the vortical structures due to the non-symmetrical serration ($\Lambda_2 = -0.005$).

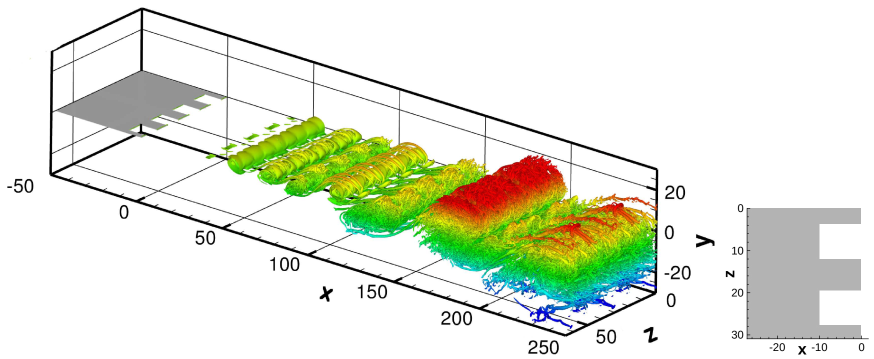

Fig. 3 Flow structures as in figure 2 but for the rectangular notch with half the spanwise extent.

The difference of the overall sound-pressure level $\Delta L_p = 20 \cdot \log_{10}(p'_{rms}/p_{ref})$ level with respect to the reference solution is shown in figures 8 a) and b) for both serrations. Directly behind the trailing edge, additional oblique waves yield an increase of pressure fluctuations by some 5 dB. On the other hand, missing large-scale

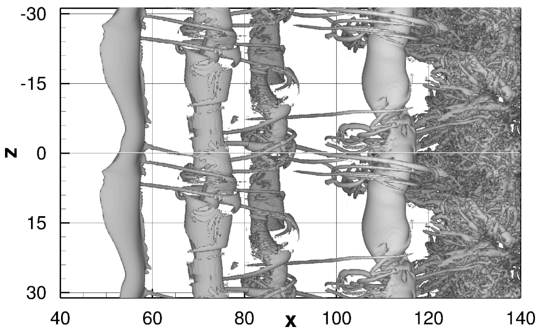

Fig. 4 Top view (snapshot) on the mixing layer downstream of the non-symmetric trailing edge along two spanwise periods. Visualization by isosurface $\Lambda_2 = -0.005$.

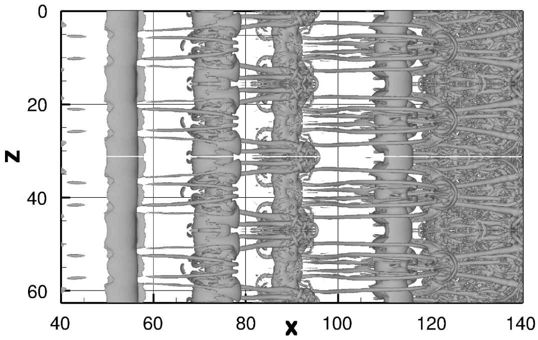

Fig. 5 Same as figure 4 but for the rectangular-notch trailing edge with half the spanwise extent.

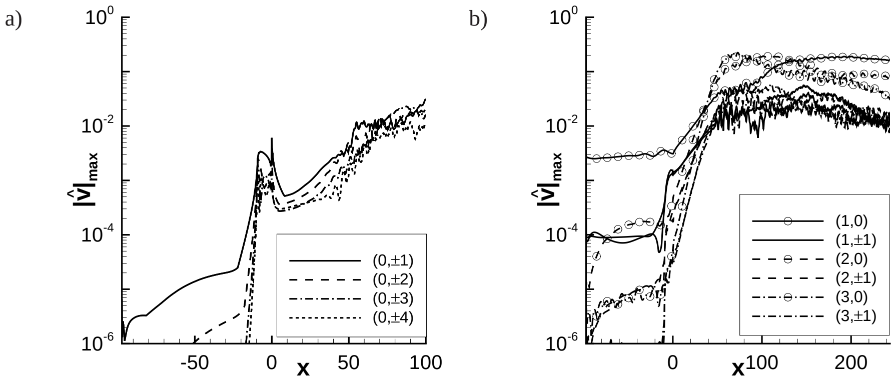

Fig. 6 Maximum wallnormal amplitudes of v for the non-symmetric serration: a) steady modes $(0,k)$, b) unsteady modes (h,k)

structures lead to lower fluctuations further downstream in the mixing layer. The reduction of the actual sound is determined by the difference in the farfield. The non-symmetric serration provides a noise reduction in the range of 6 to 10 dB. In case of the narrow rectangular notch a difference of only some 5 dB is reached. Since the result for the non-symmetric serration is quite similar to the one of the corresponding rectangular serration [1], it seems that the actual shape of the notch is of

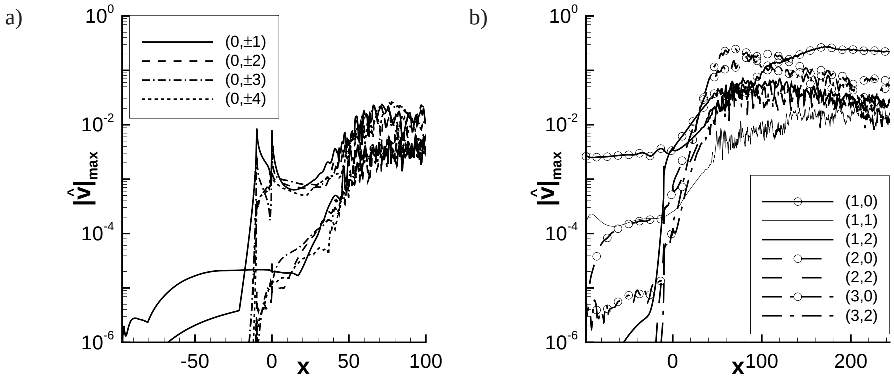

Fig. 7 Maximum amplitudes of v for the rectangular serration with half the spanwise extent $(2 \cdot \gamma_0)$: a) steady modes $(0,k)$, b) unsteady modes (h,k)

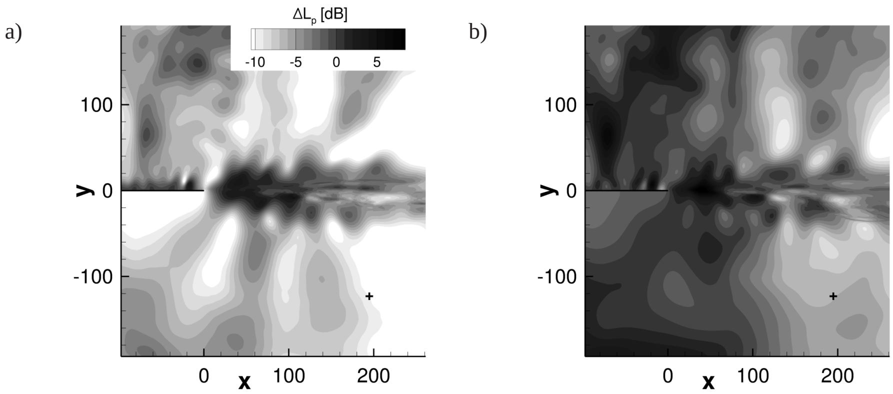

Fig. 8 Reduction of the overall sound-pressure level in dB compared to the reference case: a) non-symmetric serration and b) narrow rectangular serration, respectively. The location of the pressure spectra in figure 9 is marked by +.

minor importance. The dominant parameter of the serration is its spanwise extent / wavenumber γ as found also in the experiment by Bridges & Brown [2].

For a closer look the pressure spectra in the slow-speed stream (location indicated by + in figures 8 a) and b)) are compared with the straight trailing edge in figure 9. A notable reduction of the dominant low-frequency noise ($\omega \leq \omega_0$) is found for both serrations which is due to the decreased level of subharmonics compared to the straight trailing edge, see [1]. However for the narrow notch, sound with some $2\omega_0$ is almost as high as for the straight trailing edge. Together with the partially increased level of $|\hat{p}|$ in the range of 4 to $20\,\omega_0$ the narrow serration yields a less efficient reduction of the emitted sound.

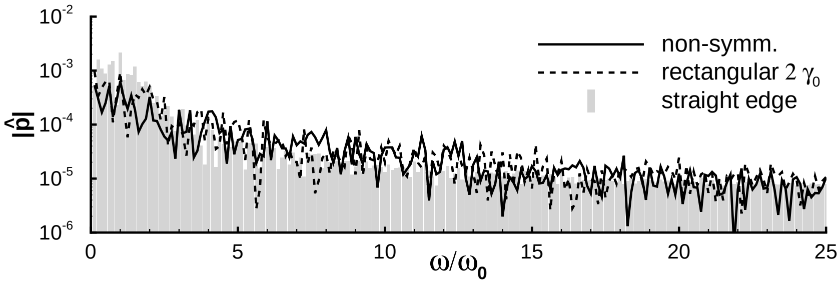

Fig. 9 Pressure spectra in the farfield $(x = 195, y = -121.8, z = 0)$ for the non-symmetric and the rectangular serration with half the spanwise extent, compared with the straight trailing edge in grey.

3 Conclusions

DNS of the mixing layer past different shapes of the trailing edge were performed. The serrations generate steady spanwise deformations according to their extent in spanwise direction. These modes interact with 2-d disturbances, generating oblique waves. While the considered flow field for the straight edge is mainly 2-d, the additional oblique modes cause a spanwise deformation of the Kelvin-Helmholtz rollers according to the shape of the serration. Subsequent streamwise vortex tubes lead to an earlier breakdown to small-scale structures. The sound emission is reduced for both geometries, however the narrow serration provides a minor noise reduction. Since the wider non-symmetric notch shows a similar emission as the corresponding rectangular one, the actual shape seems to be less important than the spanwise extent of the serration.

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